Additional Results for Jackknife Transmittance and MIS Weight Estimation

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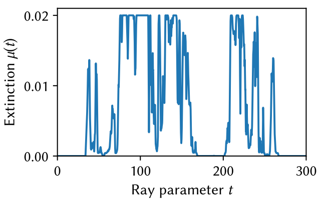
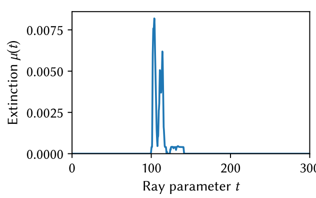
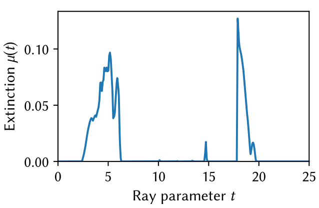


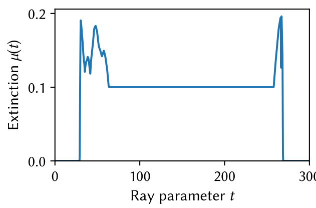
(a) Intel cloud dense



(b) Intel cloud sparse

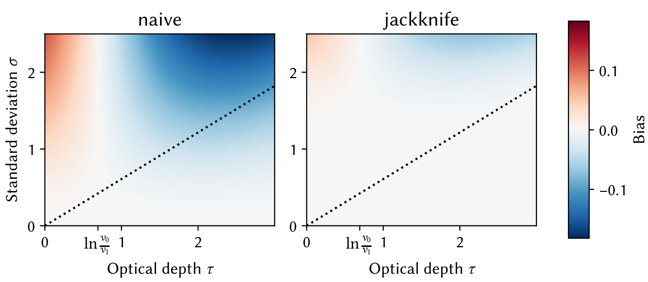


(c) Bunny cloud

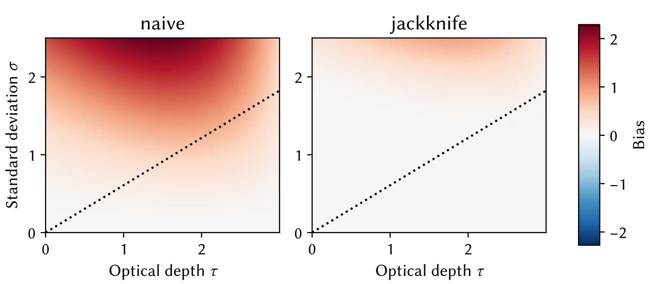


(d) Disney cloud

Fig. 1. The extinction profiles $\mu(t)$ used as examples in this supplemental document. Intel cloud dense has been used for examples in the paper.

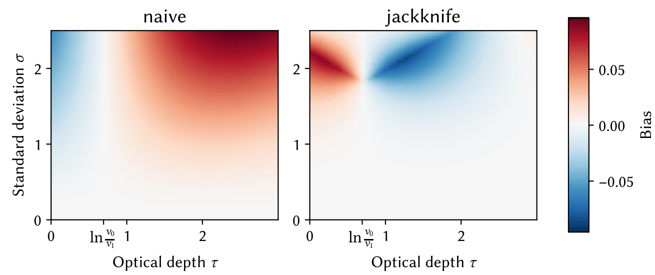


(a) $u_0 = 0, u_1 = 1, v_0 = 1, v_1 = 1/2$

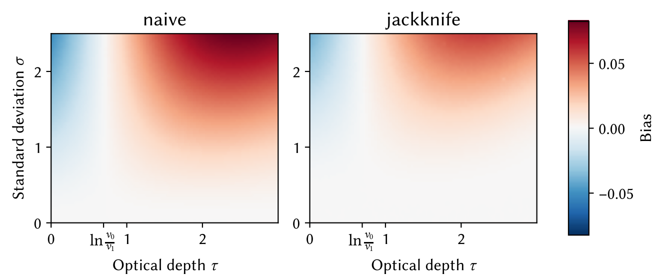


(b) $u_0 = 0, u_1 = 1, v_0 = 1, v_1 = 0.04$

Fig. 2. Additional plots of bias in MIS weight estimation for normal-distributed optical depth estimates. The jackknife estimates always have considerably lower bias than the naive estimates, especially in the region below the dashed line.



(a) Uniform distribution



(b) Laplace distribution

Fig. 3. MIS weight estimates for $(u_0, u_1, v_0, v_1) = (1, 0, 1, 1/2)$ using different distributions of optical depth estimates. Compared to the naive estimate, our jackknife estimates achieve a clear reduction in bias for these non-normal distributions in most cases.

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1 Discussion

In a few cases, the paper justifies design decisions using individual extinction profiles or parameter choices for examples. This supplemental document provides additional results where the same experiments were performed for other cases. We consider the extinction profiles $\mu(t)$ in Fig. 1. In all of these cases, the additional results support the conclusions presented in the paper. We also show results of experiments that did not fit into the paper. For details, please refer to the captions of the individual figures and tables.

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Table 1. Statistics about the accuracy of the jackknife estimate (or $\exp(-\bar{X})$ for $m = 1$) when estimating transmittance for the extinction profiles $\mu(t)$ shown in Fig. 1. In all cases, $m = 2$ estimates give the lowest bias. For the Disney cloud, the ground truth transmittance is $4.4 \cdot 10^{-12}$ and all variants succeed at estimating a near-zero transmittance. With $m = 8$ estimates, the bias is surprisingly low for the bunny cloud, but high for Intel cloud sparse, which is likely due to $N = 3$ ray marching samples being insufficient.

(a) Intel cloud dense

Estimate count m	Ray marching sample count N	Transmittance estimate mean	bias	std.
2	12	0.13411	0.00000	0.0405
3	8	0.13426	0.00015	0.0440
4	6	0.13420	0.00010	0.0528
6	4	0.13433	0.00022	0.0490
8	3	0.13485	0.00074	0.0694
1	24	0.13796	0.00385	0.0331

(b) Intel cloud sparse

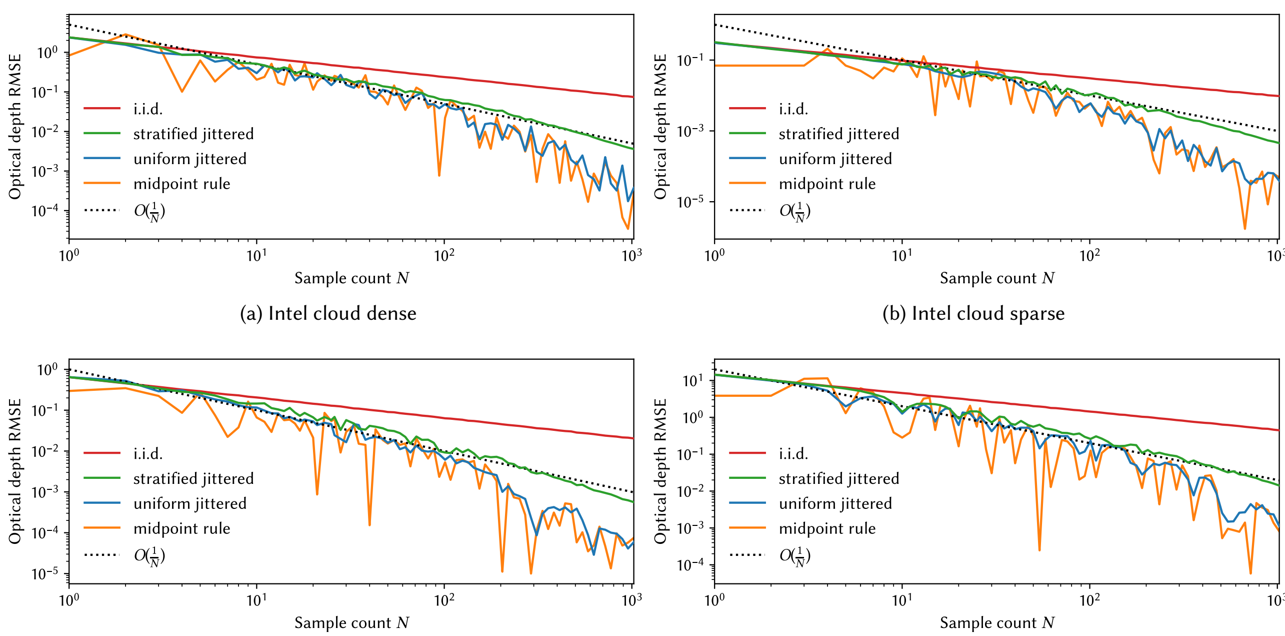
Estimate count m	Ray marching sample count N	Transmittance estimate mean	bias	std.
2	12	0.93253	0.00001	0.0423
3	8	0.93258	0.00005	0.0525
4	6	0.93256	0.00004	0.0499
6	4	0.93257	0.00005	0.0533
8	3	0.91701	-0.01552	0.1275
1	24	0.93326	0.00073	0.0367

(c) Bunny cloud

Estimate count m	Ray marching sample count N	Transmittance estimate mean	bias	std.
2	12	0.74377	0.00001	0.0597
3	8	0.74384	0.00007	0.0651
4	6	0.74386	0.00009	0.0852
6	4	0.74392	0.00016	0.0870
8	3	0.74376	-0.00001	0.0924
1	24	0.74456	0.00079	0.0347

(d) Disney cloud

Estimate count m	Ray marching sample count N	Transmittance estimate mean	bias	std.
2	12	0.00000	0.00000	0.0000
3	8	0.00000	0.00000	0.0000
4	6	0.00000	0.00000	0.0000
6	4	0.00000	0.00000	0.0000
8	3	0.00000	0.00000	0.0000
1	24	0.00000	0.00000	0.0000



(a) Intel cloud dense (b) Intel cloud sparse (c) Bunny cloud (d) Disney cloud

Fig. 4. Additional log-log-plots illustrating the convergence rate of various optical depth estimators for the extinction profiles shown in Fig. 1. Stratified jittered sampling always has a roughly linear convergence rate, which is close to uniform jittered sampling for less than $N = 100$ ray marching samples.

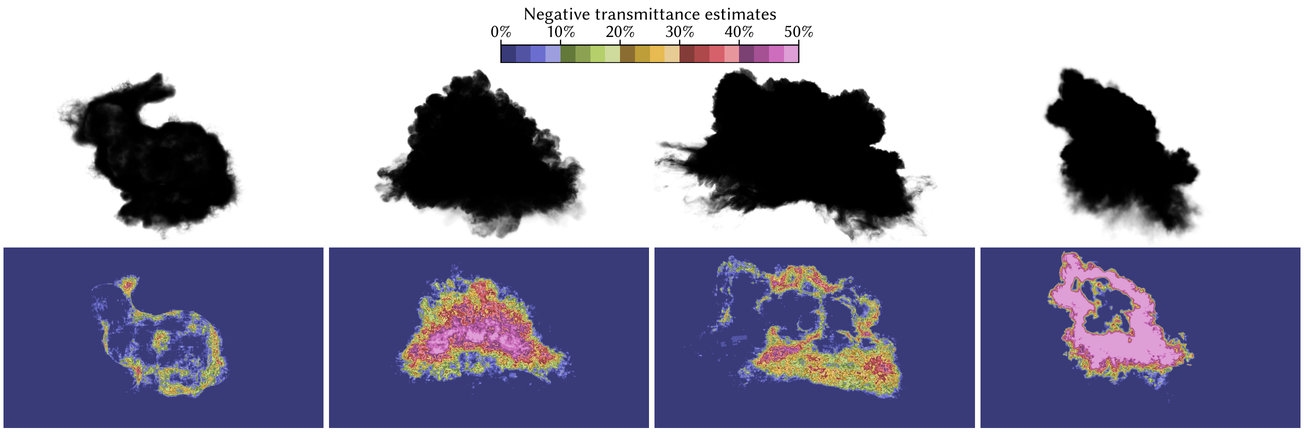


Fig. 5. An illustration of how frequently our jackknife transmittance estimate with $N = 10$ produces negative transmittance estimates (bottom row). In regions where the transmittance (top row) is clearly greater than zero, negative estimates are rare, but when the transmittance is extremely close to zero, negative estimates may occur at a rate of nearly 50%. Note that these negative estimates will still be close to zero.

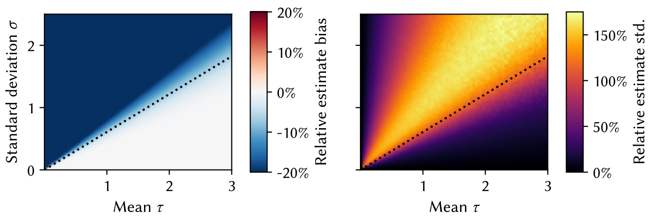


Fig. 6. Relative bias and standard deviation of a jackknife estimate of τ^{-1} based on normal-distributed X_0, X_1 with varying mean τ and standard deviation σ. If we know $X_0, X_1 \geq 0$, the relevant parts are below the dashed line, where the normal distribution has more than 95% of its mass over positive numbers. To avoid fireflies, this result uses the estimator $\frac{X_0+X_1}{\max(10^{-6}, X_0^2+X_1^2)}$ but the estimate is biased either way, since τ^{-1} is not analytic.

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